Upper bound on the cutoff in lattice electroweak theory

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ABSTRACT: We investigate numerically lattice Weinberg-Salam model without fermions for realistic values of the fine structure constant and the Weinberg angle. We also analyze the data of the previous numerical investigations of lattice Electroweak theory. We have found that moving along the line of constant physics when the lattice spacing a is decreased, one should leave the physical Higgs phase of the theory at a certain value of a. Our estimate of the minimal value of the lattice spacing is $a_c = [430 \pm 40 \text{Gev}]^{-1}$.

KEYWORDS: Higgs Physics, Spontaneous Symmetry Breaking, Lattice Gauge Field Theories, Nonperturbative Effects.

Contents

1. Introduction

In this paper we consider lattice realization of Electroweak theory (without fermions). The phase diagram of the correspondent lattice model contains physical Higgs phase, where scalar field is condensed and gauge bosons Z and W acquire their masses. This physical phase is bounded by the phase transition surface. Crossing this surface one leaves the physical phase and enters the phase of the lattice theory that has nothing to do with the conventional continuum Electroweak theory. In the physical phase of the theory the Electroweak symmetry is broken spontaneously while in the unphysical phase the Electroweak symmetry is not broken. Thus the unphysical phase is called also symmetric phase while the Higgs phase is called broken phase of the theory.

In lattice theory the ultraviolet cutoff is finite and is equal to the inverse lattice spacing: $\Lambda = \frac{1}{a}$, where a is the lattice spacing. Alternatively, the Ultraviolet cutoff in lattice theory can be defined as the momentum $\tilde{\Lambda} = \frac{\pi}{a}$ (see, for example, [1]). Later we shall imply the first definition of the cutoff.

The physical scale can be fixed, for example, using the value of the Z-boson mass $M_Z^{\mathrm{phys}} \sim 90\,\mathrm{GeV}$. Therefore the lattice spacing is evaluated to be $a \sim [90\,\mathrm{GeV}]^{-1} M_Z$, where M_Z is the Z boson mass in lattice units. Within the physical phase of the theory the lines of constant physics (LCP) are defined that correspond to constant renormalized physical couplings (the fine structure constant α, the Weinberg angle θ_W, and Higgs mass to Z-boson mass ratio $\eta = M_H/M_Z$). The points on LCP are parametrized by the lattice spacing. In general, there are two possibilities: either LCP correspondent to realistic

values of α, θ_W, and η, remains inside the given phase when a is decreased, or it crosses the boundary at a certain value of $a = a_c$. In the second case $\Lambda_c = \frac{1}{a_c}$ is the maximal possible ultraviolet cutoff in the lattice Electroweak theory.

We investigate numerically lattice realization of Weinberg-Salam model. Also we analyze existing data of the numerical investigation of the SU(2) Gauge - Higgs model. We find the indications that there exists the maximal possible ultraviolet cutoff Λ_c. Our estimate is $\Lambda_c = \frac{1}{a_c} = 430 \pm 40$ Gev. (With the definition $\tilde{\Lambda}_c = \frac{\pi}{a_c}$ we arrive at the value of the cutoff $\tilde{\Lambda}_c = 430\pi \sim 1.3$ Tev.) It is important to compare this result with the limitations on the Ultraviolet Cutoff, that come from the perturbation theory.

First, from the point of view of perturbation theory the energy scale $1\,$TeV appears in the Hierarchy problem [2]. Namely, the mass parameter μ^2 for the scalar field receives a quadratically divergent contribution in one loop. Therefore, the initial mass parameter ($\mu^2 = -\lambda_c v^2$, where v is the vacuum average of the scalar field) should be set to infinity in such a way that the renormalized mass μ_R^2 remains negative and finite. This is the content of the so-called fine tuning. It is commonly believed that this fine tuning is not natural [2] and, therefore, one should set up the finite ultraviolet cutoff Λ. From the requirement that the one-loop contribution to μ^2 is less than $10|\mu_R^2|$ one derives that $\Lambda \sim 1\,$TeV. However, strictly speaking, the possibility that the mentioned fine tuning takes place is not excluded.

In the perturbation theory there is also more solid limitation on the Ultraviolet cutoff. It appears as a consequence of the triviality problem, which is related to Landau pole in scalar field self coupling λ and in the fine structure constant α. The Landau pole in fine structure constant is related to the fermion loops and, therefore, has no direct connection with our lattice result (we neglect dynamical fermions in our consideration). Due to the Landau pole the renormalized λ is zero, and the only way to keep it equal to its measured value is to impose the limitation on the cutoff. That's why the Electroweak theory is usually thought of as a finite cutoff theory. For small Higgs masses (less than about 350 Gev) the correspondent energy scale calculated within the perturbation theory is much larger, than 1 Tev. The consideration, however, becomes nontrivial when $\lambda \to \infty$, and the perturbation expansion in λ cannot be used. In this case Higgs mass approaches its absolute upper bound,[1] and both triviality and Hierarchy scales approach each other.

2. Lattice Weinberg-Salam model

Below we use the following lattice variables:

1. The gauge field $\mathcal{U} = (U, \theta)$, where

$$U = \begin{pmatrix} U^{11} & U^{12} \\ -[U^{12}]^* & [U^{11}]^* \end{pmatrix} \in \mathrm{SU}(2), \quad e^{i\theta} \in \mathrm{U}(1), \tag{2.1}$$

   realized as link variables.

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[1]According to the previous investigations of the SU(2) Gauge - Higgs model this upper bound cannot exceed $10M_W$.

2. A scalar doublet

$$\Phi_\alpha, \ \alpha = 1, 2. \tag{2.2}$$

The action can be considered in the following form

$$S = \beta \sum_{\text{plaquettes}} \left(\left(1 - \frac{1}{2} \operatorname{Tr} U_p \right) + \frac{1}{\operatorname{tg}^2 \theta_W} (1 - \cos \theta_p) \right) +$$
$$- \gamma \sum_{xy} Re(\Phi^+ U_{xy} e^{i\theta_{xy}} \Phi) + \sum_x \left(|\Phi_x|^2 + \lambda(|\Phi_x|^2 - 1)^2 \right), \tag{2.3}$$

where the plaquette variables are defined as $U_p = U_{xy} U_{yz} U_{wz}^* U_{xw}^*$, and $\theta_p = \theta_{xy} + \theta_{yz} - \theta_{wz} - \theta_{xw}$ for the plaquette composed of the vertices x, y, z, w. Here λ is the scalar self coupling, and $\gamma = 2\kappa$, where κ corresponds to the constant used in the investigations of the SU(2) gauge Higgs model. θ_W is the Weinberg angle. Bare fine structure constant α is expressed through β and θ_W as

$$\alpha = \frac{\operatorname{tg}^2 \theta_W}{\pi \beta (1 + \operatorname{tg}^2 \theta_W)}. \tag{2.4}$$

The renormalized Weinberg angle is to be calculated through the ratio of the lattice masses: $\cos \theta_W = M_W / M_Z$. The renormalized fine structure constant can be extracted through the potential for the infinitely heavy external charged particles.

Lattice model with the action (2.3) was investigated numerically in the number of papers. Most of the papers dealt with the SU(2) Gauge-Higgs model, i.e. with the case $\theta_W = \pi/2$. The system with arbitrary θ_W has been investigated numerically at unphysically large α in [3]. Here we list some of the papers that consider SU(2) Gauge - Higgs model at realistic values of β around $\beta = 8$: [4–17]. Implying that the hypercharge field is to be included into consideration perturbatively, one can use expression (2.4) with $\sin^2\theta_W = 0.23$ and estimate $\alpha = \frac{1}{110}$ that is not far from its physical value $\alpha(M_W) = \frac{1}{128}$.

3. Numerical investigation of the model at $\theta_W = \pi/6$

Here we report the results of our numerical investigation of the system (2.3) for $\theta_W = \frac{\pi}{6}$ (corresponds to $\sin^2\theta_W = 0.25$), $\lambda \to \infty$, and renormalized α around $\alpha(M_W) = \frac{1}{128}$. From the very beginning we fix the unitary gauge $\Phi_1 = const.$, $\Phi_2 = 0$.

The following variables are considered as creating a Z boson and a W boson, respectively:

$$Z_{xy} = Z_x^\mu = \sin[\operatorname{Arg}U_{xy}^{11} + \theta_{xy}],$$
$$W_{xy} = W_x^\mu = U_{xy}^{12} e^{-i\theta_{xy}}. \tag{3.1}$$

Here, μ represents the direction (xy).

After fixing the unitary gauge the electromagnetic U(1) symmetry remains:

$$U_{xy} \to g_x^\dagger U_{xy} g_y,$$
$$\theta_{xy} \to \theta_{xy} - \alpha_y/2 + \alpha_x/2, \tag{3.2}$$

where $g_x = \mathrm{diag}(e^{i\alpha_x/2}, e^{-i\alpha_x/2})$. There exists a U(1) lattice gauge field, which is defined as

$$A_{xy} = A_x^\mu = [-\mathrm{Arg}U_{xy}^{11} + \theta_{xy}] \bmod 2\pi \tag{3.3}$$

that transforms as $A_{xy} \to A_{xy} - \alpha_y + \alpha_x$. The field W transforms as $W_{xy} \to W_{xy}e^{-i\alpha_x}$.

The W boson field is charged with respect to the U(1) symmetry. Therefore we fix the lattice Landau gauge in order to investigate the W boson propagator. The lattice Landau gauge is fixed via minimizing (with respect to the U(1) gauge transformations) the following functional:

$$F = \sum_{xy}(1 - \cos(A_{xy})). \tag{3.4}$$

Then we extract the mass of the W boson from the correlator

$$\frac{1}{N^6} \sum_{\bar{x},\bar{y}} \left\langle \sum_\mu W_x^\mu (W_y^\mu)^\dagger \right\rangle \sim e^{-M_W|x_0-y_0|} + e^{-M_W(L-|x_0-y_0|)} \tag{3.5}$$

Here the summation $\sum_{\bar{x},\bar{y}}$ is over the three "space" components of the four-vectors x and y while x_0, y_0 denote their "time" components. N is the lattice length in "space" direction. L is the lattice length in the "time" direction.

The Z-boson mass is calculated using the correlator

$$\frac{1}{N^6} \sum_{\bar{x},\bar{y}} \left\langle \sum_\mu Z_x^\mu Z_y^\mu \right\rangle \sim e^{-M_Z|x_0-y_0|} + e^{-M_Z(L-|x_0-y_0|)} \tag{3.6}$$

It is worth mentioning, that in the Z-boson channel many photon state also exists. The mass of the correspondent state on the finite lattice we used is, however, larger than that of the Z-boson itself. For example, on the lattice $16^3 \times 24$ the minimal mass of the 3-photon state is $M_{3\gamma} = 2\frac{2\pi}{16} + \frac{4\pi}{16} \sim 1.5$. Moreover, from the point of view of perturbation theory this state appears in the correlator (3.6) through the virtual loop and is suppressed by the factor α^3.

In order to evaluate the mass of the Higgs boson we use the correlator [18]:

$$\sum_{\bar{x},\bar{y}}\langle H_x H_y \rangle \sim e^{-M_H|x_0-y_0|} + e^{-M_H(L-|x_0-y_0|)} + const, \tag{3.7}$$

and the following operators that create Higgs bosons:

$$H_V^x = \sum_y Re(U_{xy}^{11}e^{i\theta_{xy}}); \qquad H_W^x = \sum_y |W_{xy}|^2; \qquad H_Z^x = \sum_y Z_{xy}^2 \tag{3.8}$$

Here H_V^x, H_W^x, H_Z^x are defined at the site x, the sum $\sum_y$ is over its neighboring sites y.

We perform the calculation of renormalized fine structure constant α_R using the potential for infinitely heavy external fermions. We consider Wilson loops for the right-handed external leptons:

$$\mathcal{W}_{\mathrm{lept}}^{\mathrm{R}}(l) = \langle \mathrm{Re}\, \Pi_{(xy)\in l} e^{2i\theta_{xy}} \rangle. \tag{3.9}$$

Here l denotes a closed contour on the lattice. We consider the following quantity constructed from the rectangular Wilson loop of size $r \times t$:

$$\mathcal{V}(r) = \log \lim_{t \to \infty} \frac{\mathcal{W}(r \times t)}{\mathcal{W}(r \times (t+1))}. \tag{3.10}$$

Due to exchange by virtual photons at large enough distances we expect the appearance of the Coulomb interaction

$$\mathcal{V}(r) = -\frac{\alpha_R}{r} + const. \tag{3.11}$$

It should be mentioned here, that in order to extract the renormalized value of α one may apply to $\mathcal{V}$ the fit obtained using the Coulomb interaction in momentum space. The lattice Fourier transform then gives

$$\mathcal{V}(r) = -\alpha_R \mathcal{U}(r) + const,$$
$$\mathcal{U}(r) = \frac{\pi}{N^3} \sum_{\bar{p} \neq 0} \frac{e^{ip_3 r}}{\sin^2 p_1/2 + \sin^2 p_2/2 + \sin^2 p_3/2} \tag{3.12}$$

Here N is the lattice size, $p_i = \frac{2\pi}{L} k_i, k_i = 0, \ldots, L-1$. On large enough lattices at $r \ll L$ both definitions approach each other. For example, for $L = 75, r \in [1, 10]$ the linear fit to the dependence $\mathcal{U}(r)$ on $\frac{1}{r}$ gives $\mathcal{U}(r) \sim 0.97/r - 0.18$. However, on the lattices of sizes we used the difference is important. Say, on the lattice 16^3 the fit is $\mathcal{U}(r) \sim 0.71/r - 0.4$ (for $r \in [1,5]$). Thus, the values of the renormalized α_R extracted from (3.11) and (3.12) are essentially different from each other. Any of the two ways, (3.11) or (3.12), may be considered as the *definition* of the renormalized α on the finite lattice. And there is no particular reason to prefer the potential defined using the lattice Fourier transform of the Coulomb law in momentum space. Actually, our study shows that the single $1/r$ fit approximates $\mathcal{V}$ much better. Therefore, we used it to extract α_R. This should be compared with the results of [17], where for similar reasons the single $e^{-\mu r}/r$ fit (instead of the lattice Yukawa fit) was used in order to determine the renormalized coupling constant in the SU(2) Gauge Higgs model.

In figure 1 we present the phase diagram for the lattice model in the β - γ plane. Mainly we used lattices of sizes 16^4. Some results were checked on the lattices of size 24^4. For the evaluation of masses we used lattices $6^3 \times 12$, $8^3 \times 16$, $12^3 \times 24$, and $16^3 \times 24$. At small values of β this system was considered in [3]. The dotted vertical line on the left side of the figure represents the deconfinement phase transition corresponding to the U(1) constituents of the model. The continuous line corresponds to the transition between the broken and the symmetric phases of the model. Physical Higgs phase of the system is situated in the right upper corner of figure 1.

The dotted vertical line on the right-hand side of the diagram represents the line, where the renormalized α (calculated on the lattice 16^4) is constant and is close to its physical value $\frac{1}{128}$. Actually, on the tree level this would be the straight line $\beta = \frac{\text{tg}^2\theta_W}{\pi\alpha(1+\text{tg}^2\theta_W)} \sim 10$. According to our numerical results on the lattice 16^4 at $\gamma = 1$ and β close to $\beta = 15$ the renormalized α_R is equal to $\frac{1}{128\pm1}$. In addition we checked our results on the renormalized
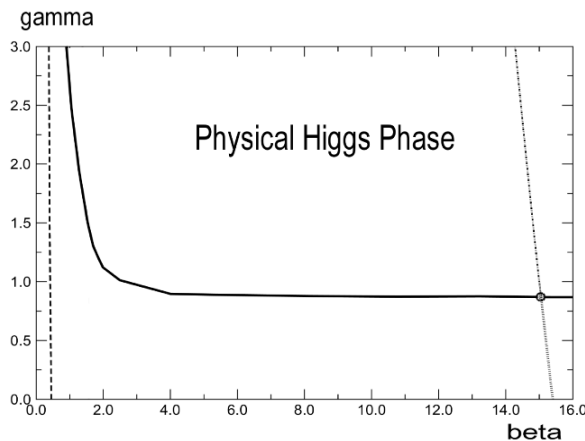
Figure 1: The phase diagram of the lattice model at fixed λ in the (β, γ)-plane.

α_R on the lattice $12^3 \times 24$. We have found on this lattice the same value of α_R (within the statistical errors) as on the lattice 16^4. So, we conclude that the renormalized fine structure constant calculated using our choice of the fit for the potential is not sensitive to the change of the lattice size. The given line of constant renormalized α_R is almost the straight line that is defined by the two points: $[\gamma = 1; \beta = 15]$ and $[\gamma = 1.5; \beta = 14.81]$. The accuracy of the calculation of α_R is around 1%.

The position of the phase transition lines on this figure was localized, mainly, using methods developed in [19, 20]. In particular, we considered the behavior of various monopole-like topological defects that exist in the given model. (For the definition of the correspondent monopole currents, their density and percolation probability, see [19].) The densities and percolation probabilities of the constructed monopole currents appear to be very sensitive to the phase transitions. Say, the monopole currents constructed of the field θ feel the deconfinement phase transition corresponding to the U(1) constituents of the model. Their worldlines are extracted from the hypercharge field θ in the following way:

$$j_Y = \frac{1}{2\pi} {}^*d([d2\theta]\mathrm{mod}2\pi) \tag{3.13}$$

(Here we used the notations of differential forms on the lattice. For their definition see [19, 20] and references therein.) The monopole density is defined as

$$\rho = \left\langle \frac{\sum_{\mathrm{links}} |j_{\mathrm{link}}|}{4L \times N^3} \right\rangle, \tag{3.14}$$

where N is the lattice size in "space" direction, L is the lattice size in "time" direction in lattice units. (We often used asymmetric lattices for the calculation of the variables related to the monopole properties.) The density of hypercharge monopoles is nonzero within the confinement-like phase and falls sharply within the deconfinement phase. The average action of the model appears to be inhomogeneous in the small vicinity of the phase transition line.

The monopole currents constructed of the field A (in a way similar to (3.13)) feel the transition between the broken and the symmetric phases of the model: $j_A =$
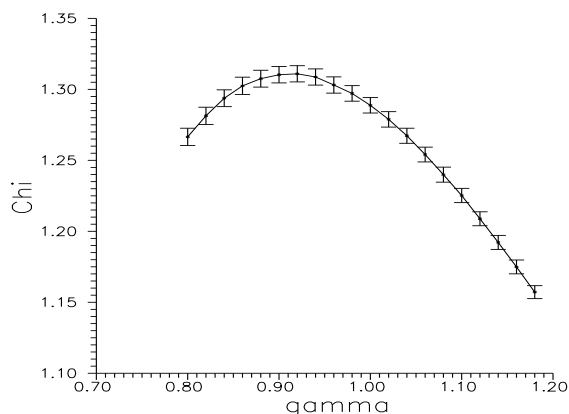
Figure 2: Susceptibility $\chi = \langle H_Z^2 \rangle - \langle H_Z \rangle^2$ at $\beta = 15$ on the lattice $8^3 \times 16$.

$\frac{1}{2\pi} {}^*d([dA] \bmod 2\pi)$. Their density drops in the physical Higgs phase. In order to investigate topological defects extracted from the Z-boson field we use the definition of the Z-boson creation operator different from (3.1):

$$Z'_{xy} = [\text{Arg} U_{xy}^{11} + \theta_{xy}] \bmod 2\pi, \tag{3.15}$$

Then we investigate monopole currents constructed of the field Z': $j_Z = \frac{1}{2\pi} {}^*d([dZ'] \bmod 2\pi)$. Their density also drops in the physical Higgs phase.

In order to localize the position of this transition we also use the susceptibility $\chi = \langle H_Z^2 \rangle - \langle H_Z \rangle^2$. In figure 2 the dependence of the susceptibility on γ on the lattice $8^3 \times 16$ is represented at fixed $\beta = 15$. H_Z is composed of Z field according to expression (3.8). We also check our data represented on figure 2 using the lattices $12^3 \times 24$ and 16^4. We do not find any dependence of χ on the lattice size.

It can be seen that the maximum of the susceptibility composed of H_Z corresponds to the values of γ around $\gamma = 0.92$. We found that the percolation probabilities of both monopole currents extracted from the fields A and Z' vanish at the same value of γ. In summary, we evaluate the position of the transition between the two phases at $\beta = 15$ as $\gamma_c = 0.92 \pm 0.02$.

It is worth mentioning that according to our numerical results monopoles extracted from the fields A and Z' are condensed in the unphysical symmetric phase of the model. The correspondent field configurations carry magnetic charge and dominate in the vacuum of the symmetric phase. Therefore, this phase indeed has nothing to do with the real continuum physics.

The behavior of the densities of the considered topological objects is in general very similar to that of the $\text{SU}(2) \times \text{U}(1)/Z_2$ model investigated in [19]. It is worth mentioning that the line of the transition between the broken and the symmetric phases of the model can actually be a crossover line. In general we evaluate error bars in determination of the phase transition points given in figure 1 as $\Delta\gamma = \pm 0.05; \Delta\beta = \pm 0.05$ although in some regions of the phase diagram the accuracy is better.
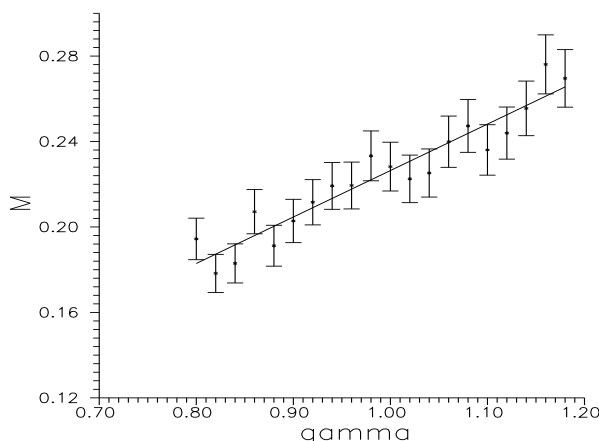
Figure 3: M_Z as a function of γ.

For the calculation of the W-boson and Z-boson masses we used lattices of sizes $6^3 \times 12$, $8^3 \times 16$, $12^3 \times 24$, and $16^3 \times 24$. It has been found that the W-boson mass contains an artificial dependence on the lattice size. We suppose, that this dependence is due to the photon cloud surrounding the W-boson. The energy of this cloud is related to the renormalization of the fine structure constant. It has been shown above that the definition of renormalized α_R is ambiguous on the finite lattice. The difference between the two possible definitions (via the single $1/r$ fit and via the lattice Coulomb potential) depends strongly on the lattice size. On the other hand, the Z-boson correlator does not possess this artificial dependence on the lattice size. Therefore, we use the Z-boson mass in order to fix Ultraviolet cutoff in the model.

Careful investigation of the ZZ correlator at the point $\gamma = 1, \beta = 15$ shows that M_Z does not depend on the lattice size. The value of mass $M_Z = 0.22 \pm 0.01$ at $\gamma = 1, \beta = 15$ was obtained on four different lattices of sizes $6^3 \times 12$, $8^3 \times 16$, $12^3 \times 24$, and $16^3 \times 24$. The dependence of the Z-boson mass on γ at $\beta = 15$ on the lattice $8^3 \times 24$ together with the linear fit are given in figure 3. The linear fit is $M_Z = 0.009 + 0.217\gamma$.

Basing on this data we conclude that the Z-boson mass in lattice units in the physical Higgs phase of the theory cannot exceed the value 0.21 ± 0.01 for $\beta = 15$ as we locate the transition between the two phases at $\gamma = 0.92 \pm 0.02$. At the point $[\beta = 15, \gamma = 0.92]$ the value of renormalized α_R does not deviate much from the value calculated on the line $\alpha_R = \frac{1}{128}$. Actually, the deviation is within 1%. Thus we expect the maximal possible Ultraviolet cutoff at realistic value of the fine structure constant cannot exceed $\Lambda_c = 430 \pm 40$ Gev.[2] So, the Ultraviolet cutoff grows when γ is decreased, its maximal value within the physical Higgs phase is achieved at the transition point and cannot exceed $\Lambda_c = \frac{1}{a_c} = 430 \pm 40$ Gev (or, $\tilde{\Lambda}_c = \frac{\pi}{a_c} \sim 1.35$ Tev).

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[2]We also like to notice here that in the previously investigated SU(2) Gauge - Higgs model it was found that the gauge boson mass in lattice units grows when one moves into the physical Higgs phase starting from the transition point (when the gauge coupling β is fixed [4 – 17].)

As for the Higgs boson mass, due to the insufficient statistics we cannot extract M_H from our data with reasonable accuracy. According to our (very rough) estimate at $\beta = 15, \gamma \in [0.8; 1.2]$ we have $M_H/M_Z \sim 9 \pm 2$. This estimate is in agreement with the investigation of the SU(2) Gauge Higgs model [15 – 17] performed near the transition point for the London limit of the Higgs potential and realistic β. Actually, as in [15] we made our estimate based on the consideration of the correlator for small space-time separation (≤ 3). It was found in [17] that at larger distances the second mass parameter close to $2M_W$ contributes to the correlator. In [17] in order to evaluate Higgs boson mass in this situation this second value was considered as the mass of the bound state of the two gauge bosons, and only the first mass in the given channel was interpreted as the Higgs boson mass.

4. The tree level estimates of lattice quantities

At finite λ the line of constant renormalized α is not a line of constant physics, because the mass of the Higgs boson depends on the position on this line. Thus, in order to investigate the line of constant physics one should vary λ together with γ to keep the ratio of lattice masses M_H/M_W constant.

In order to obtain the tree level estimates let us rewrite the lattice action in an appropriate way. Namely, we define the scalar field $\tilde{\Phi} = \sqrt{\frac{\gamma}{2}}\Phi$. We have:

$$S = \beta \sum_{\text{plaquettes}} \left(\left(1 - \frac{1}{2}\operatorname{Tr}U_p \right) + \frac{1}{\operatorname{tg}^2\theta_W}(1 - \cos\theta_p) \right) + $$
$$+ \sum_{xy} |\tilde{\Phi}_x - U_{xy}e^{i\theta_{xy}}\tilde{\Phi}_y|^2 + \sum_x \left(\mu^2|\tilde{\Phi}_x|^2 + \tilde{\lambda}|\tilde{\Phi}_x|^4 \right) + \omega, \quad (4.1)$$

where $\mu^2 = -2(4 + (2\lambda - 1)/\gamma)$, $\tilde{\lambda} = 4\frac{\lambda}{\gamma^2}$, and $\omega = \lambda V$. Here $V = L^4$ is the lattice volume, and L is the lattice size.

For negative μ^2 we fix Unitary gauge $\tilde{\Phi}_2 = 0$, $\operatorname{Im}\tilde{\Phi}_1 = 0$, and introduce the vacuum value of $\tilde{\Phi}$: $v = \frac{|\mu|}{\sqrt{2\tilde{\lambda}}}$. We also introduce the scalar field σ instead of $\tilde{\Phi}$: $\tilde{\Phi}_1 = v + \sigma$. We denote $V_{xy} = (U_{xy}^{11}e^{i\theta_{xy}} - 1)$, and obtain:

$$S = \beta \sum_{\text{plaquettes}} \left(\left(1 - \frac{1}{2}\operatorname{Tr}U_p \right) + \frac{1}{\operatorname{tg}^2\theta_W}(1 - \cos\theta_p) \right) + $$
$$+ \sum_{xy} ((\sigma_x - \sigma_y)^2 + |V_{xy}|^2 v^2) + \sum_x 2|\mu|^2\sigma_x^2$$
$$+ \sum_{xy} \left((\sigma_y^2 + 2v\sigma_y)|V_{xy}|^2 - 2(\sigma_x - \sigma_y)\operatorname{Re}V_{xy}(\sigma_y + v) \right) + $$
$$+ \sum_x \tilde{\lambda}\sigma_x^2(\sigma_x^2 + 4v\sigma_x) + \tilde{\omega}, \quad (4.2)$$

where $\tilde{\omega} = \omega - \tilde{\lambda}v^4 V$.

Now we easily derive the tree level estimates:

$$M_H = \sqrt{2}|\mu| = 2\sqrt{4 + (2\lambda - 1)/\gamma};$$

$$M_W = \sqrt{2}\frac{v}{\sqrt{\beta}} = \sqrt{\frac{\gamma(4\gamma + 2\lambda - 1)}{2\lambda\beta}};$$

$$M_W = \cos\theta_W M_Z$$

$$M_H/M_W = \sqrt{8\lambda\beta/\gamma^2};$$

$$\Lambda = \sqrt{\frac{2\lambda\beta}{\gamma(4\gamma + 2\lambda - 1)}}\,[80\,\text{GeV}]; \tag{4.3}$$

the fine structure constant is given by the formula (2.4) and does not depend on λ and γ. From (4.3) we learn that at the tree level LCP on the phase diagram corresponds to fixed $\beta = \frac{\text{tg}^2\theta_W}{\pi\alpha(1+\text{tg}^2\theta_W)} \sim 10$ and $\eta = M_H/M_W$, and is given by the equation $\lambda(\gamma) = \frac{\eta^2}{8\beta}\gamma^2$. Actually, numerical research shows that the real LCP stays not far from this tree level estimate (for $\lambda \ll 1$).

The important case is $\lambda = \infty$, where the tree level estimates give

$$M_H = \infty;$$

$$M_W = \sqrt{\frac{\gamma}{\beta}};$$

$$M_Z = \sqrt{\frac{\gamma}{\beta}}\cos^{-1}\theta_W;$$

$$\Lambda = \sqrt{\frac{\beta}{\gamma}}\,[80\,\text{GeV}]; \tag{4.4}$$

In the SU(2) gauge Higgs model for the small values of $\lambda \ll 0.1$ the tree level estimate for M_H/M_W gives values that differ from the renormalized ratio by about 20%[14]. The tree level estimate for the ultraviolet cutoff is about 310 GeV at $\lambda = \infty, \gamma = 1, \beta = 15$ that is not far from the numerical result given in the previous section. In the SU(2) Gauge Higgs model at $\lambda = \infty$ the critical $\gamma_c = 0.63$ for $\beta = 8$ [17]. At this point the tree level estimate gives $\Lambda = 285$ Gev while the direct measurements give $\Lambda \in [270; 470]$ Gev for values of $\gamma \in [0.64; 0.95]$ [17]. The investigations of the SU(2) Gauge Higgs model showed that a consideration of finite λ does not change much the estimate for the gauge boson mass. However, at finite λ and values of γ close to the phase-transition point the tree level formula does not work at all.

The tree level estimate for the critical γ is $\gamma_c = (1 - 2\lambda)/4$. At small λ this formula gives values that are close to the ones obtained by the numerical simulations [15–17]. In particular, $\gamma_c \to 0.25$ ($\kappa_c \to 0.125$) at $\lambda \ll 1$. However, this formula clearly does not work for $\lambda > 1/2$. From [18, 15–17] we know that the critical coupling in the SU(2) Gauge Higgs model is about $2 - 4$ times smaller for $\lambda = 0$ than for $\lambda = \infty$.

5. Analysis of the existing data

From the previous research we know that the phase diagram in the β - γ plane of the lattice SU(2) Gauge-Higgs for any fixed λ resembles the phase diagram represented in the figure 1. The only difference is that in the SU(2) Gauge-Higgs model the confinement-deconfinement phase transition corresponding to the U(1) constituents of the model is absent. The direct measurement of the renormalized coupling β_R shows [4−17] that the line of constant renormalized coupling constant (with the value close to the experimental one) intersects the phase transition line. Also we know from the direct measurements of M_W in the SU(2) Gauge-Higgs model that the ultraviolet cutoff is increased when one is moving along this line from the physical Higgs phase to symmetric phase. It is also worth mentioning that the line of the transition between the broken and the symmetric phases of the model can actually be a crossover line. .

According to (4.3) the W-boson mass in lattice units vanishes at the critical $\gamma_c = (1 - 2\lambda)/4$. This means that the tree level estimate predicts the appearance of an infinite ultraviolet cutoff at the transition point for finite λ. At infinite λ the tree level estimate gives nonzero values of lattice M_W for any nonzero γ. Our numerical investigation of SU(2)⊗U(1) model (at infinite λ) and previous calculations in the SU(2) Gauge Higgs model (both at finite λ and at $\lambda = \infty$) showed that for the considered lattice sizes renormalized masses do not vanish and the transition is either of the first order or a crossover. (Actually, the situation, when the cutoff tends to infinity at the position of the transition point means that there is a second order phase transition.) The dependence on the lattice sizes for the SU(2) Gauge Higgs model was investigated, for example, in [13]. Namely, for $\beta = 8$, $\lambda \sim 0.00116$, where $M_H \sim M_W$, the correlation lengths were evaluated at the critical value $\kappa_c = \gamma_c/2$. For different lattice sizes (from $12^3 \times 28$ to $18^3 \times 36$) no change in correlation length was observed [13].

In the table we summarize the data on the ultraviolet cutoff $\frac{1}{a}$ achieved in selected lattice studies of the SU(2) Gauge Higgs model. (a is the lattice spacing.) Everywhere β is around $\beta \sim 8$ and the renormalized fine structure constant is around $\alpha \sim 1/110$.

Among the papers listed in this table there are results of both finite temperature and zero temperature studies. However, in the case when the finite temperature simulations are performed the authors either refer to the analogous simulations of the zero temperature theory or performed such simulations directly. This is related to the fact that the only way to set up the scale in the theory and, correspondingly, to calculate the temperature, is to deal with the zero temperature model on the symmetric lattice. To be explicit, one should calculate lattice spacing a on the symmetric lattice via calculation of the gauge boson mass. Then on the asymmetric lattice (with the same values of couplings as on the symmetric one) the value of temperature is $1/(Na)$, where N is the lattice size in time direction. The ultraviolet cutoffs used in the mentioned lattice studies of the finite temperature theory actually correspond to the zero temperature models, where these values have been calculated.

Strictly speaking, the above described picture works at infinite (or, high enough) lattice size. If $T \to 0$, then one should use lattice with the time extent $N_T = \frac{1}{Ta} \to \infty$. That's

Reference	Ultraviolet Cutoff $\frac{1}{a}$ (GeV)	M_H (GeV)
[4]	140 (space direction) 570 (time direction)	80
[5]	280 (time direction)	80
[6]	280	34
[7]	110	16
[8]	90 (space direction) 350 (time direction)	34
[9]	280	48
[10]	140	35
[11]	280	20 , 50
[12]	190	50
[13]	260	57 - 85
[14]	200 - 300	47 - 108
[15]	400	480
[16]	330 - 470	280 - 720
[17]	250 - 470	720 ($\lambda = \infty$)

Table 1: Reported values of the cutoff in lattice Electroweak theory.

why the value of lattice spacing calculated on the ideal infinite symmetric lattice is to be used in the finite temperature study at small enough temperatures. Our analysis shows, that the smallest value of a is around $[400\text{Gev}]^{-1}$ (see section 7 of the present paper). Our study shows also, that M_Z does not depend on the lattice size L for $L > 5$. Thus for the time extent of the asymmetric lattice $N_T > 5$ corresponding to $T < 80$ Gev the value of a calculated on the symmetric lattice can be applied. However, already at the temperatures of the order of 400 Gev it is necessary to use lattice with the time extent $N = \frac{1}{Ta} \sim 1$. Therefore, it is obvious, that at $T > 80$ Gev the lattice theory suffers from lattice artifacts. At the temperatures larger, than 400 Gev, it cannot be applied in principle.

In principle, the effect of lattice artifacts could be partially corrected if the effective value of lattice spacing is used that is different from that of calculated on the symmetric lattice. If so, the effective upper bound on the Ultraviolet cutoff $\Lambda_c = \frac{1}{a_c}$ can be considered as depending on temperature. However, the discussion of such a dependence is out of the scope of the present paper.

6. Triviality problem and the hierarchy scale

The emergence of the triviality problem in lattice theory was considered in a number of papers (see, for example, [16, 17]). According to the common view on the problem the renormalized λ tends to zero when the ultraviolet cutoff tends to infinity. Thus at the infinite value of the cutoff Higgs sector becomes trivial (noninteracting). As a result the renormalized ratio M_H/M_W should tend to zero when the cutoff tends to infinity while the other renormalized couplings (α and θ_W) remain constant. However, at finite ultraviolet cutoff this ratio may remain far from zero. In the situation, when the measured Higgs

boson mass is larger than the inverse lattice spacing, we cannot consider the Higgs boson as a real quantum state existing in the theory. (We do not think, however, that in this situation the theory looses sense at all.) Thus, when M_H becomes of the order of the cutoff, it approaches its absolute upper bound. This gives the so-called triviality upper bound on the Higgs mass allowed in lattice Electroweak theory. According to the previous investigation of the SU(2) Gauge-Higgs model this triviality bound is $M_H/M_W < 10$ (see, for example, [16, 17]).

Basing on the perturbative treatment of the triviality problem one expects that in the lattice theory this problem appears as follows. Each Line of Constant Physics (correspondent to fixed renormalized α, θ_W, and M_H/M_W) must be ended at a certain value of the cutoff related to the triviality problem. Basing on the perturbation theory one may expect, that this value of the cutoff for $M_H < 350$ Gev is larger, than 10 Tev (see, for example, [22] and references therein). If M_H approaches its absolute upper bound M_H^c, then the perturbation theory predicts decrease of the maximal Ultraviolet cutoff Λ_t related to the triviality problem.

The Hierarchy scale is around 1 Tev. So, if the mentioned above picture is valid, moving along the Line of Constant Physics at $M_H < 350$ Gev we would encounter the Hierarchy scale much earlier, than the triviality problem. However, as it will be explained in the next section, there are indications that the Line of Constant Physics always stops at the point, where the value of the ultraviolet cutoff Λ_c is at the Hierarchy scale. This means that within the lattice theory the emergence of the triviality problem is more complicated, than it was usually thought. We suppose, that both Landau pole in scalar self coupling, and the Hierarchy problem in perturbation theory, as well as the appearance of the maximal cutoff $\Lambda_c \sim 1$ Tev in the lattice theory may actually be the manifestations of the same phenomenon.

It is worth mentioning, that if $M_H \to M_H^c$, then perturbative Λ_t is decreased and approaches the value of the Higgs mass. So, Λ_t and Λ_c approach each other (see figure 2 of [22]).

7. The maximal value of the cutoff

On the lattice the bare mass parameter in lattice units is $\mu^2 = -2(4 + (2\lambda - 1)/\gamma)$. In the lattice theory we reach the point where the renormalized μ_R^2 becomes positive, if we are moving along the line of constant α, while the ultraviolet cutoff Λ is increased. This is the point of a phase transition between the broken and the symmetric phases of the model.

The content of the fine tuning in continuum approach is that we set up the initial parameter μ^2 in such a way that the quadratically divergent contribution to μ_R^2 is cancelled. This means that $-\mu^2$ should be as large as const $\times \Lambda^2$. In the perturbation theory, in principle, for any given Λ we can choose an appropriate value of μ^2. Therefore the naive guess would be that on the lattice in order to increase the cutoff the value of bare lattice λ should be increased (then $-\mu^2 = 2(4 + (2\lambda - 1)/\gamma)$ is increased). In our simulations we used the maximal possible value of λ, i.e. $\lambda = \infty$. And we have found that the value of the cutoff cannot exceed its maximal value Λ_c. At infinite λ the tree level estimate

gives $\Lambda_c^{\text{tree}} = \sqrt{\frac{\beta}{\gamma_c}}[80\,\text{Gev}]^{-1}$. If we substitute $\gamma_c \sim 1$ and $\beta \sim 15$ then the tree level estimate gives $\Lambda_c^{\text{tree}} \sim 310$ Gev. Our calculations[3] gave us value $\Lambda_c = \frac{1}{a_c} \sim 430 \pm 40$ for $\sin^2\theta_W = 0.25$, $\alpha_R \sim \frac{1}{128}$. (Here a_c is the value of lattice spacing.) In the SU(2) Gauge-Higgs model the maximal reported value of $\Lambda = \frac{1}{a}$ is 470 Gev. It is worth mentioning here that the weak coupling expansion in lattice theory [21] gives the prediction that the maximal possible ultraviolet cutoff is achieved in lattice Electroweak theory at infinite λ. The value 470 Gev was obtained when the U(1) constituent of the model was neglected. Moreover, the fine structure constant in the correspondent research was around $\frac{1}{110}$. In our research the U(1) subgroup of the Electroweak gauge group is taken into account and α_R is around its physical value $\frac{1}{128}$. That's why we feel it appropriate to estimate the maximal cutoff in the lattice Electroweak theory (with dynamical fermions neglected) equal to the value calculated in our work.

Thus basing on our data and on the data of the previous numerical research we expect that Λ_c remains finite at the transition point for any λ. If so, then in the lattice theory there is no way to avoid entering the wrong phase while increasing Λ with *any* choice of initial parameters of the model. However, the possibility still remains that the second order phase transition between the symmetric and the broken phases may appear at selected exceptional values of the coupling constants. Then at these points the Ultraviolet cutoff may become infinite.

8. Conclusions and discussion

To conclude, in this paper we reported the results of numerical investigation of the lattice Weinberg-Salam model at infinite bare scalar self coupling. We also analyzed results of the previous lattice study of SU(2) Gauge-Higgs model. Both our results and the previous data indicate that the values of lattice spacings smaller, than a critical value a_c, cannot be achieved in principle. Basing on the existing data we expect, that a_c is about $[430 \pm 40\text{Gev}]^{-1}$.

Our study shows that the susceptibility represented in figure 2 does not depend on the lattice size. This can be considered as the indication, that the transition between the Higgs phase and the symmetric phase of the model is the crossover. We also have found, that the percolation of monopole-like topological defects appears as an order parameter for this transition. That's why we conclude, that the given transition may belong to the class of the transitions of the so-called Kertesz type (see, for example, [23]).

The important question is how the minimal value a_c of the lattice spacing depends on the details of lattice regularization. In particular, one may suppose that it could become possible to find out the improved lattice action that allows to decrease a_c. However, this question is out of the scope of the present paper.

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[3]In the previous numerical investigations of lattice Electroweak theory at realistic values of β the U(1) constituent of the model was not taken into account. It was implied that the hypercharge field is to be taken into account using perturbation expansion. Thus possible nonperturbative effects were ignored. However, we see that nonperturbative effects are important for evaluation of maximal possible Ultraviolet cutoff in lattice Weinberg-Salam model (at least, at $\lambda \to \infty$).

It is the common point of view, that due to the triviality problem the Weinberg - Salam model should be considered as a finite cutoff theory. The main result of our paper is that the value of the maximal cutoff in lattice Electroweak theory is essentially smaller, than it was thought previously. Namely, we suppose, that it is about $\Lambda_c = \frac{1}{a_c} \sim 430 \pm 40$ Gev (or, $\tilde{\Lambda}_c = \frac{\pi}{a_c} \sim 1.3$ Tev). Although we neglect dynamical fermions and consider the scalar field potential in London limit, we suppose that the investigation of the theory with the finite value of scalar self coupling and with dynamical fermions included will not change our estimate crucially. Thus we expect, that the Weinberg-Salam model can be used only at the energies $\mathcal{E} \ll 1$ Tev. At the same time at the energies approaching 1 Tev the other theory should be used.[4]

The appearance of the upper bound on the cutoff in lattice Electroweak theory may have important consequences in finite temperature theory. In particular, one of the scenarios of baryon asymmetry appearance is related to Electroweak sphalerons. However, the correspondent energy scale 10 Tev is far above Λ_c. Moreover, the lattice Electroweak theory cannot be applied[5] at $T > \Lambda_c \sim 430$ Gev, because time extent of the lattice is evaluated as $N \sim \frac{1}{aT}$.

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